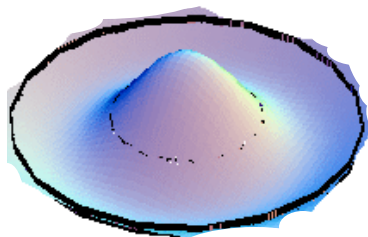


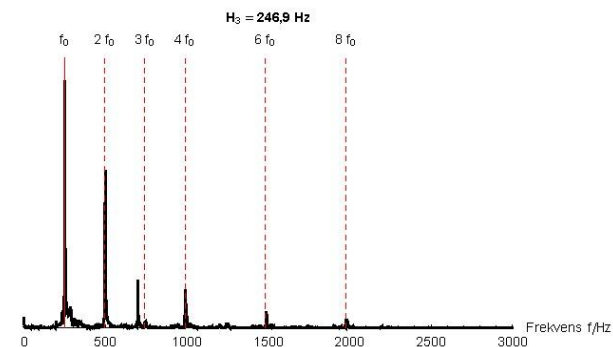
Olietøndens fysik

Hovedfagskollokvium af
Morten Gersborg-Hansen

morten @ gersborg . dk



Oprindeligt givet på
Niels Bohr Institutet
Københavns Universitet
8. juli 2004

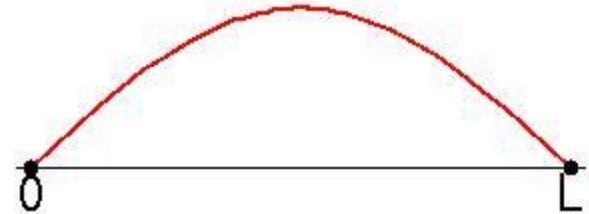


Simpelt system: 1D

Udsvinget af en snor opfylder bølgeligningen:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

$$c^2 = \frac{F}{\rho}$$



Grænsebetingelse:

$$u(0) = u(L) = 0$$

Tilladte frekvenser:

$$\omega_n = \frac{n\pi c}{L} \quad \text{dvs.} \quad f_n = \frac{\omega_n}{2\pi} = n \frac{c}{2L}$$

Normalsvingninger:

$$u_n(x, t) = (a_n \sin \omega_n t + b_n \cos \omega_n t) \sin \frac{\omega_n x}{c}$$

Harmoniske:

$$f_n = n f_1$$

Generel løsning:

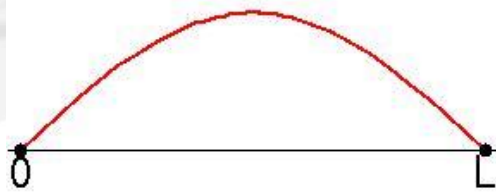
$$u(x, t) = \sum_n c_n u_n(x, t)$$

Normalsvingninger: 1D

Frekvens $f_n = nf_1$ $f_1 = \frac{c}{2L}$

Grundtone

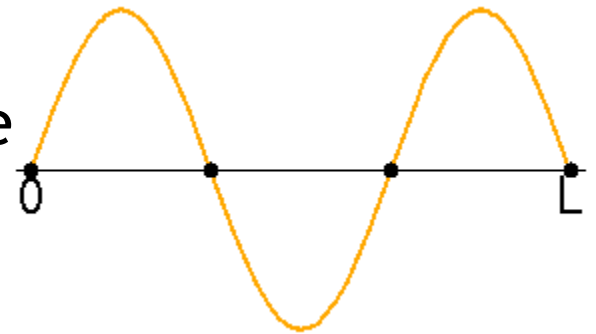
$$n = 1$$



(0)

2. overtone

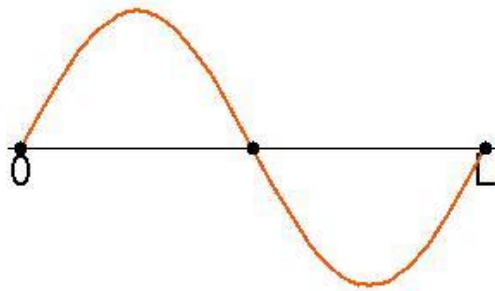
$$n = 3$$



(2)

1. overtone

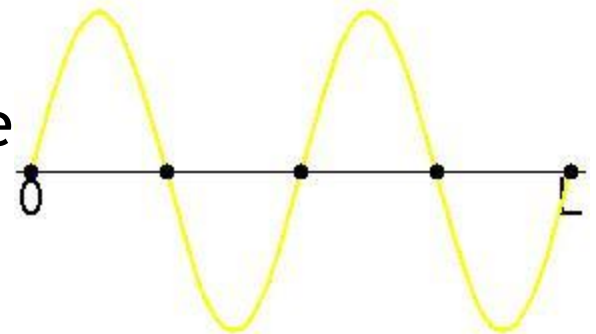
$$n = 2$$



(1)

3. overtone

$$n = 4$$



(3)

2D: Membransvingninger

Forskydningen af en fleksibel membran opfylder 2D-bølgeligningen:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \nabla^2 u$$

$$c^2 = \frac{T}{\rho}$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$$



Separation af de variable:

$$u(r, \theta, t) = F(r)G(\theta)H(t)$$

Grænsebetingelse:

$$u(R, \theta, t) = 0$$

Normalsvingninger:

$$u_{mn}(r, \theta, t) = (a_{mn} \cos ck_{mn} t + b_n \sin ck_{mn} t) J_n(k_{mn} r) \cos n\theta$$

$$k_{mn} = \frac{\alpha_{mn}}{R} \quad \alpha_{mn} : m\text{'te positive nulpunkt af } J_n(x)$$

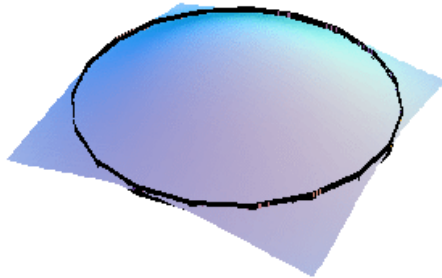
2D: Membransvingninger

$$u_{mn}(r, \theta, t) = (a_{mn} \cos ck_{mn}t + b_n \sin ck_{mn}t) J_n(k_{mn}r) \cos/\sin n\theta$$

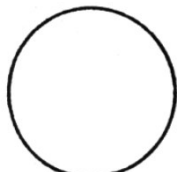
$$\text{Frekvens: } f_{(0,0)} = \frac{\alpha_{10}}{2\pi R} \sqrt{\frac{T}{\rho}} = \frac{2,405}{2\pi R} \sqrt{\frac{T}{\rho}}$$

Indicering v. antal knudelinier som (r, θ) : $(m-1, n)$

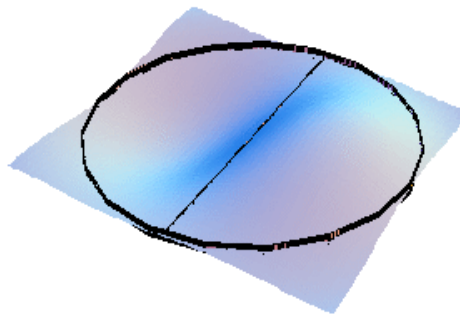
Eksempler:



$(0,0)$



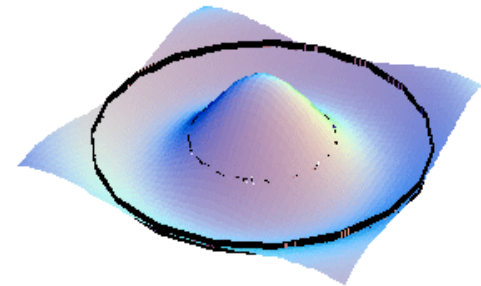
1.000



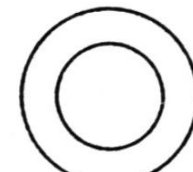
$(0,1)$



1.594



$(1,0)$

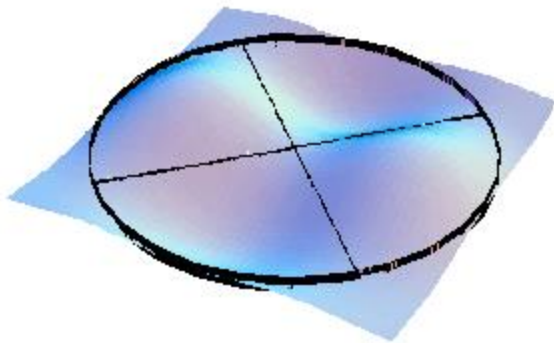


2.296

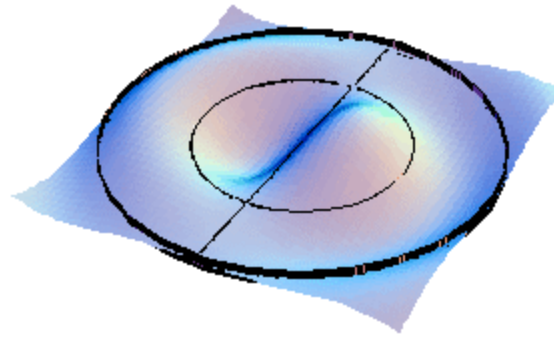
$\frac{f}{f_{(0,0)}}$

2D: Membransvingninger

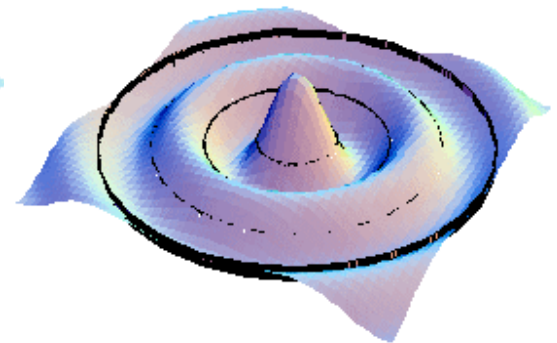
Flere eksempler:



(0,2)



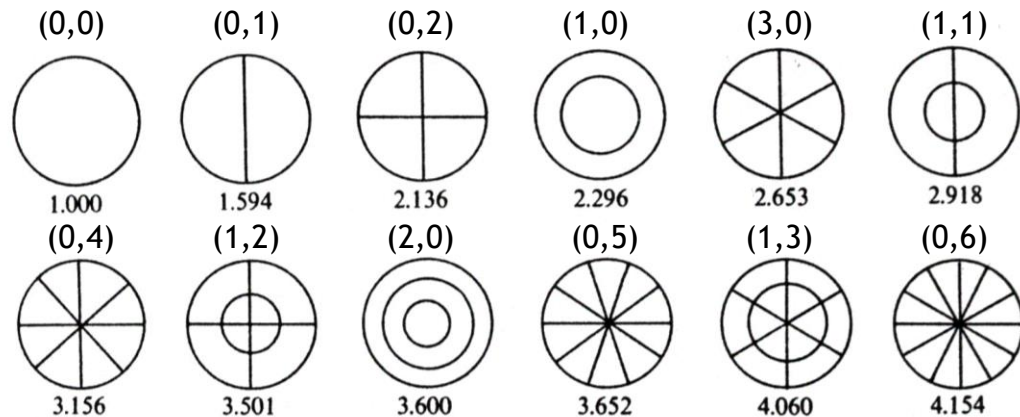
(1,1)



(3,0)

Udvalgte modes

med $\frac{f}{f_{(0,0)}}$

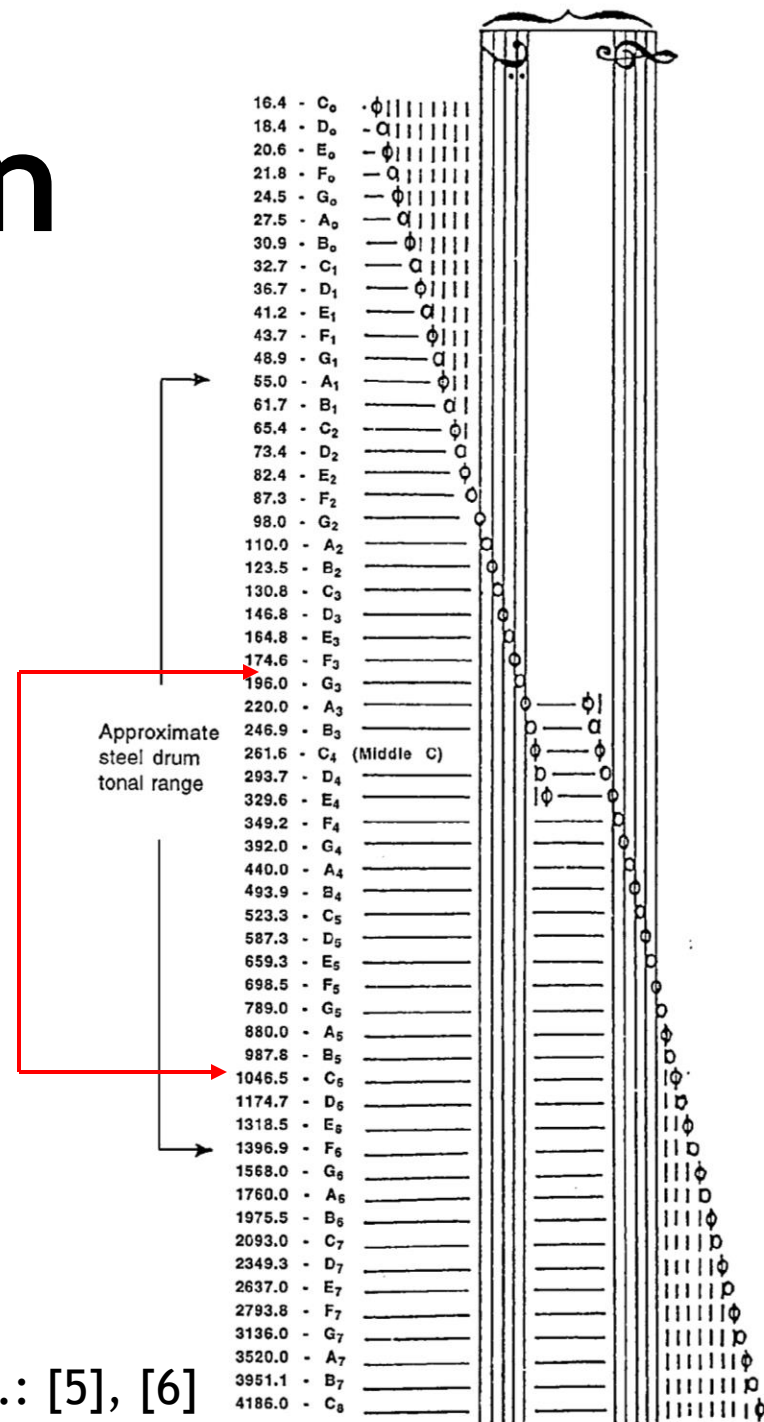
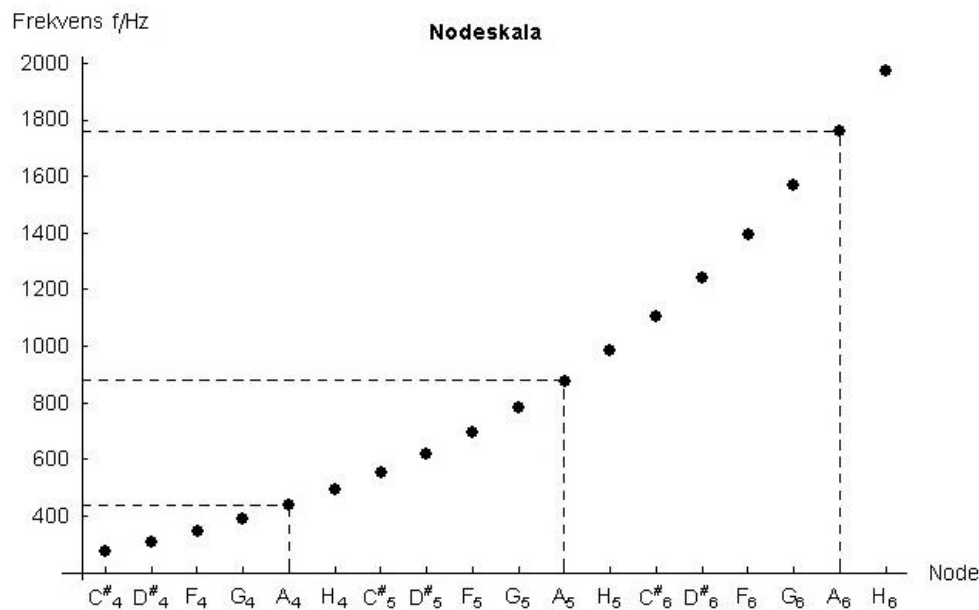


Nodeskalaen

Konstruksjon:

- $A_4 = 440 \text{ Hz}$ (kammerton)
- Oktav = dobbelt frekvens
- 12 halvtoner pr. oktav

dvs. $f = 440 \text{ Hz} \cdot \left(\sqrt[12]{2}\right)^n$

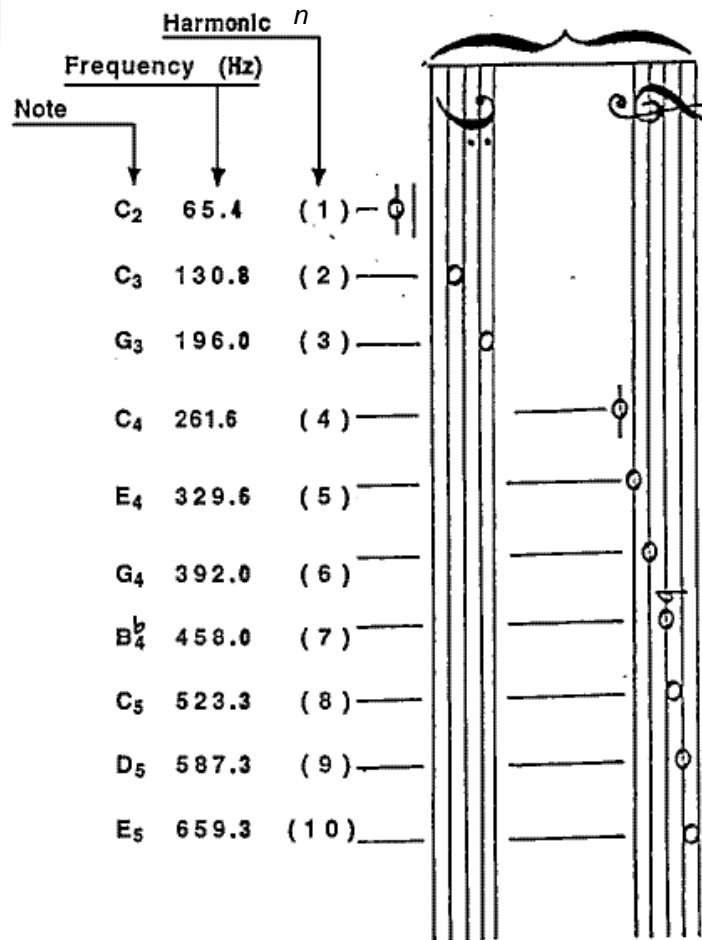


Ref.: [5], [6]

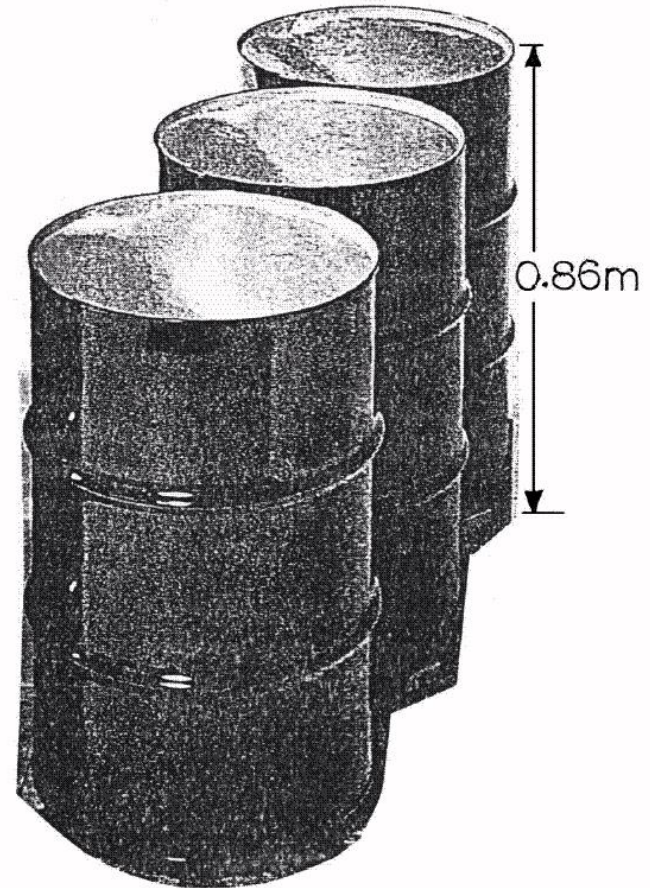
Harmoniske overtoner

Frekvenser:

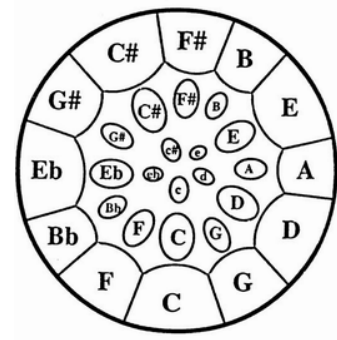
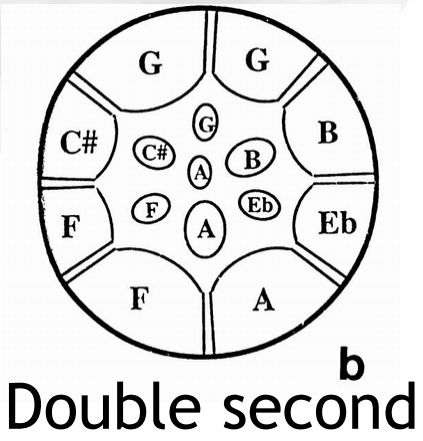
$$f_n = n f_0$$



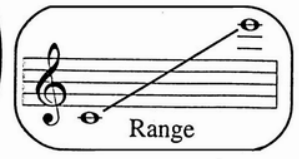
Steelpan



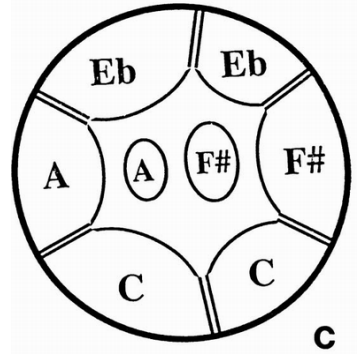
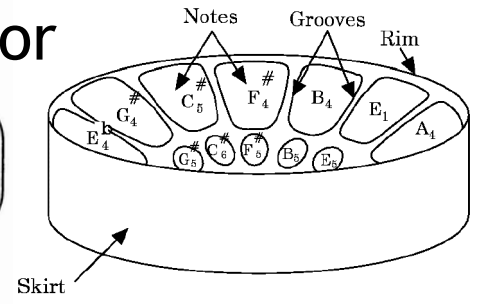
Forskellige typer



Lead/Tenor

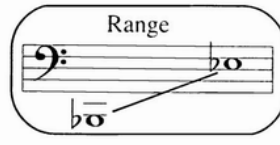
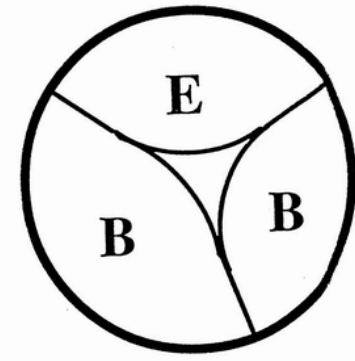


a



Cello

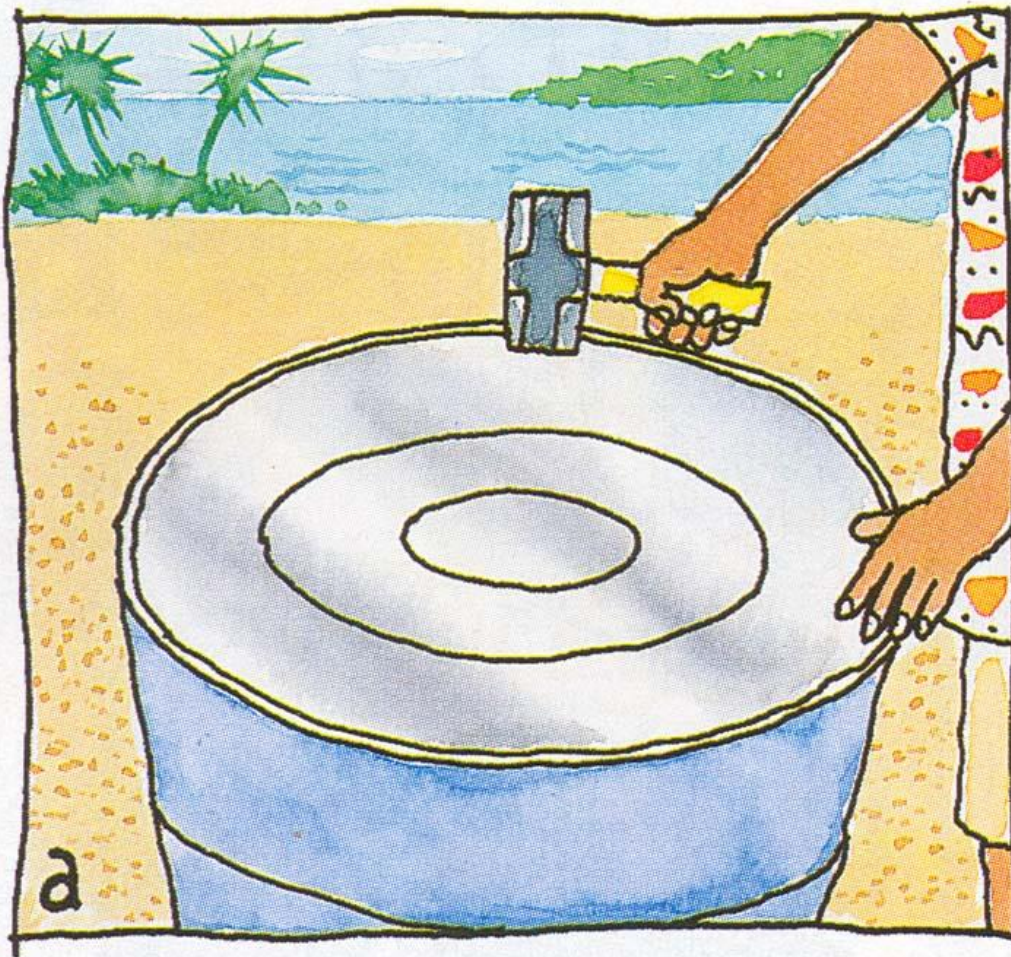
c



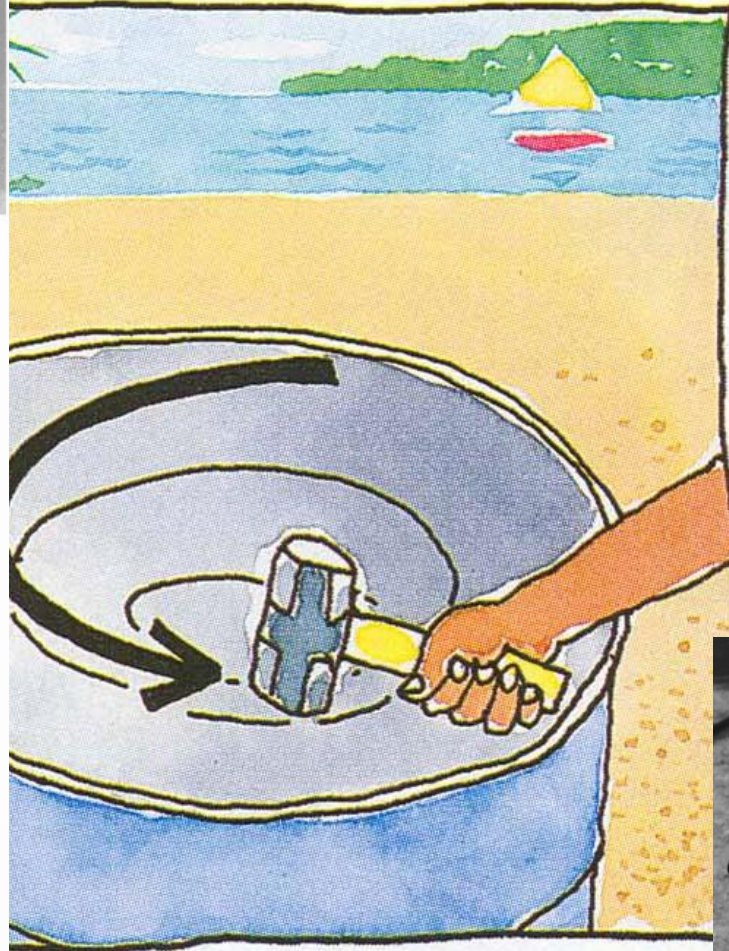
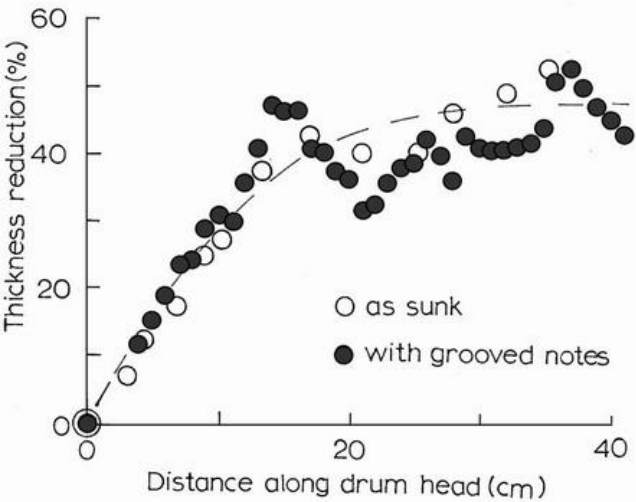
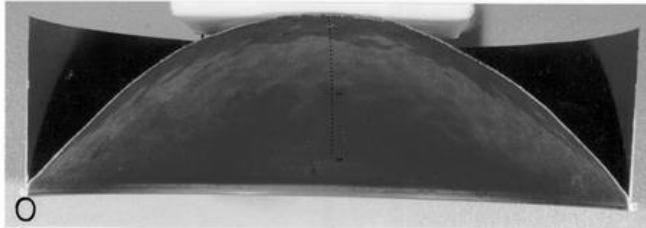
d



1. Start

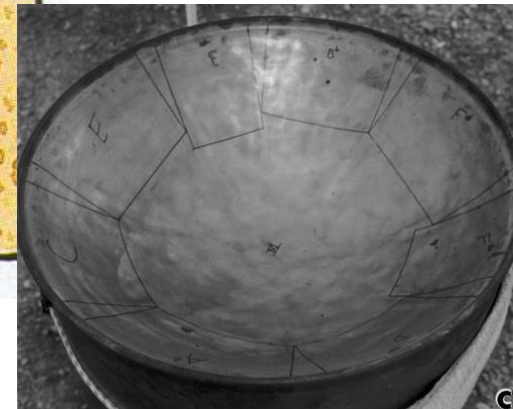
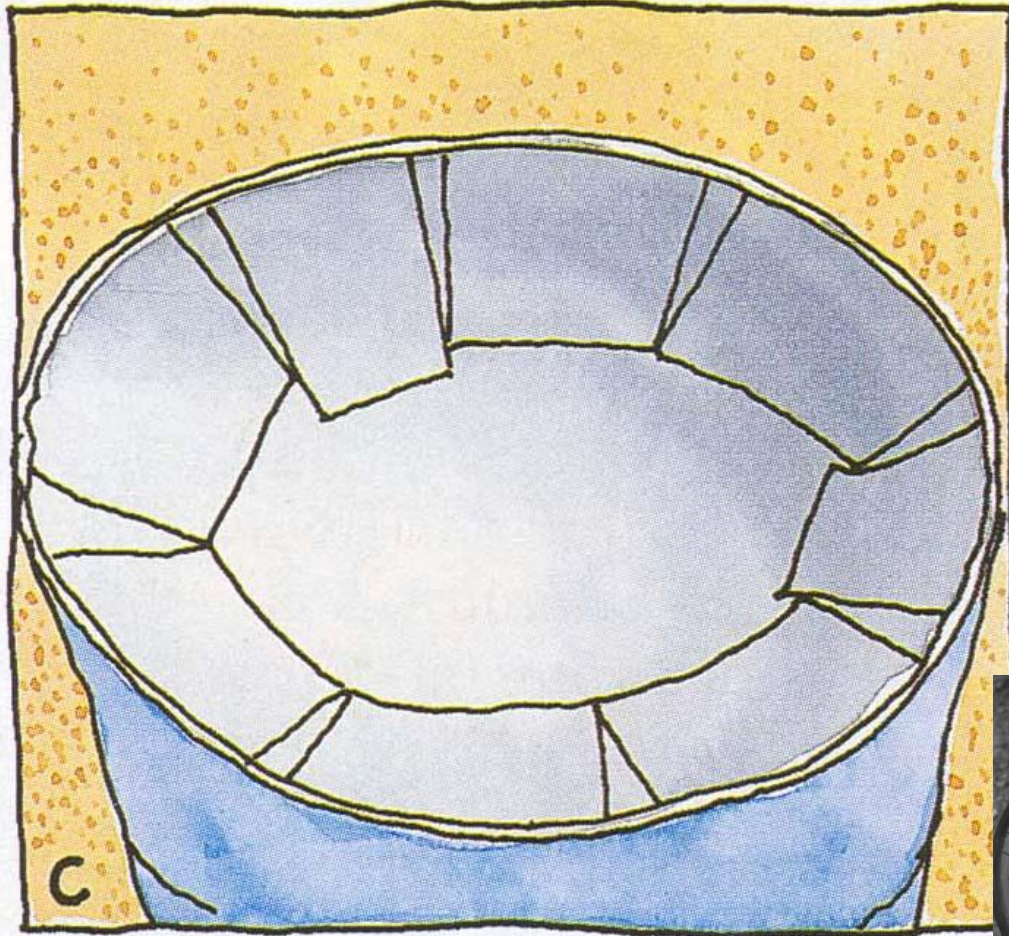


2. Sænkning



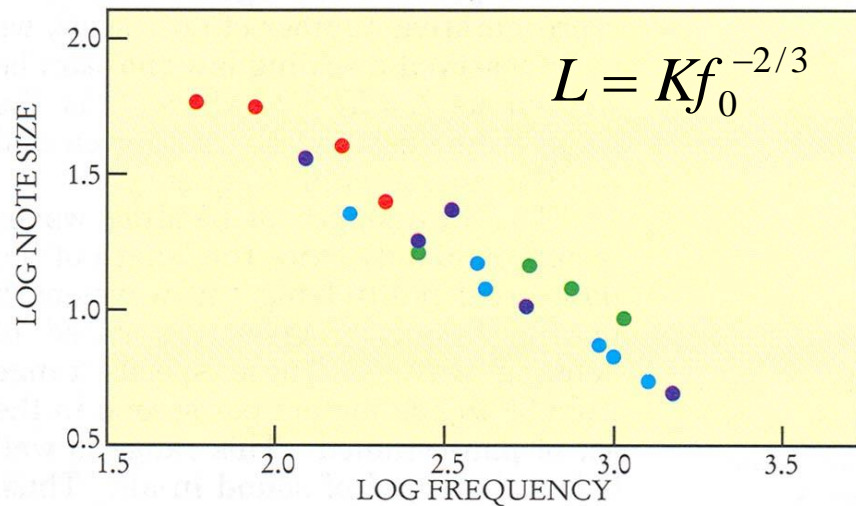
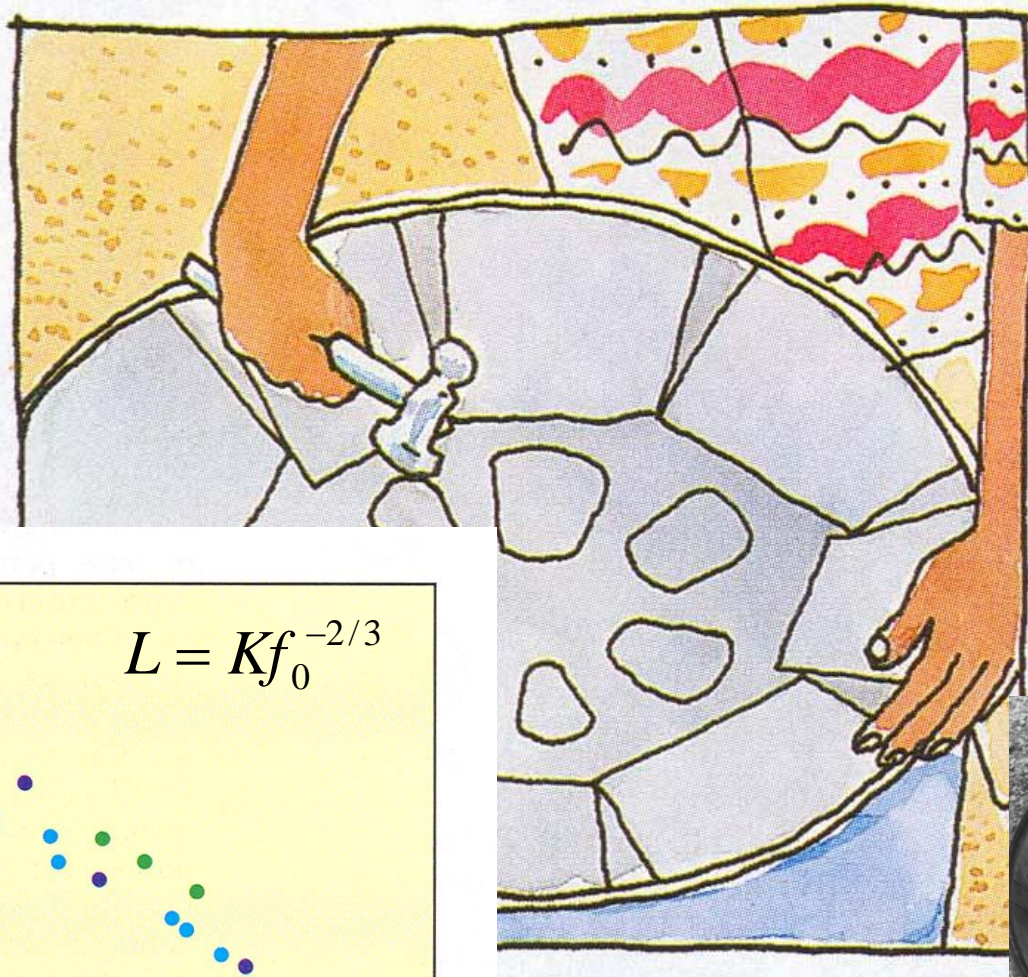
Ref.: [4], [5]

3. Nodeplacering



Ref.: [4], [5]

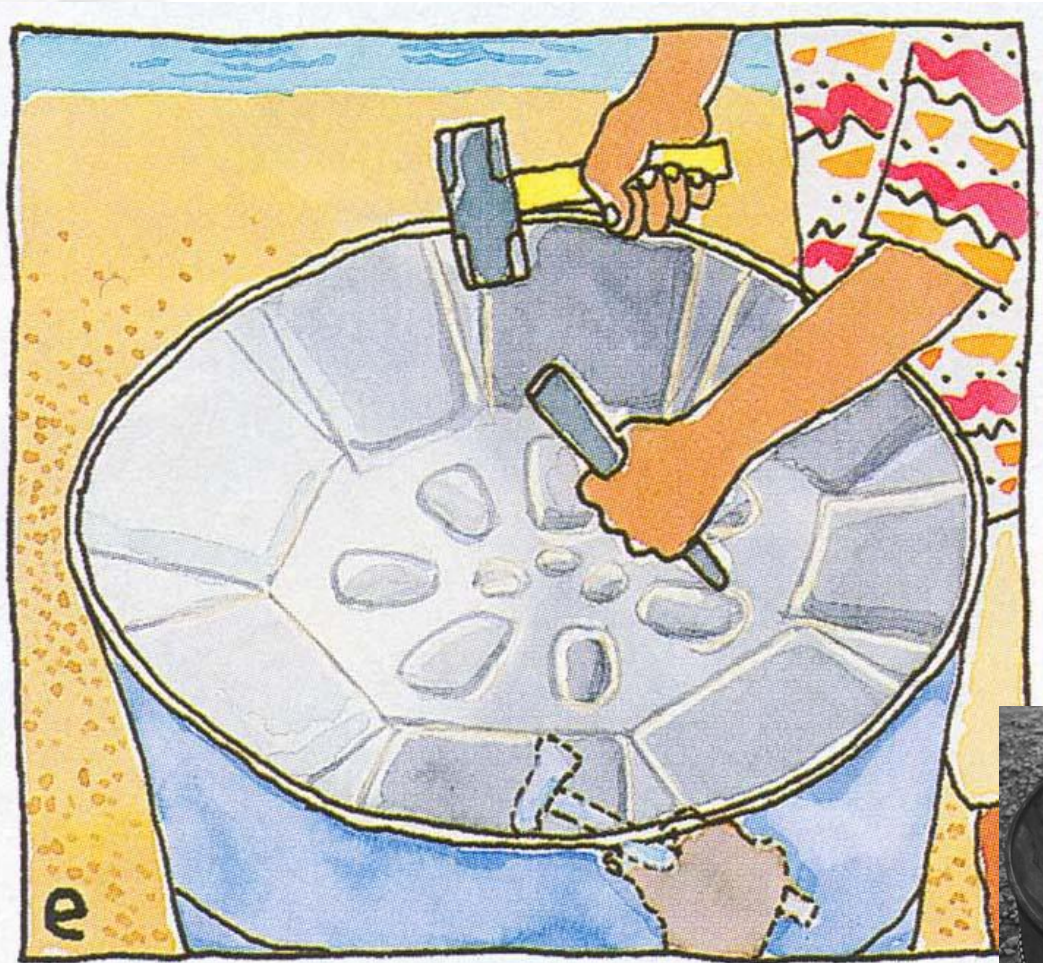
4. Nodeforming



Ref.: [3], [4], [5]

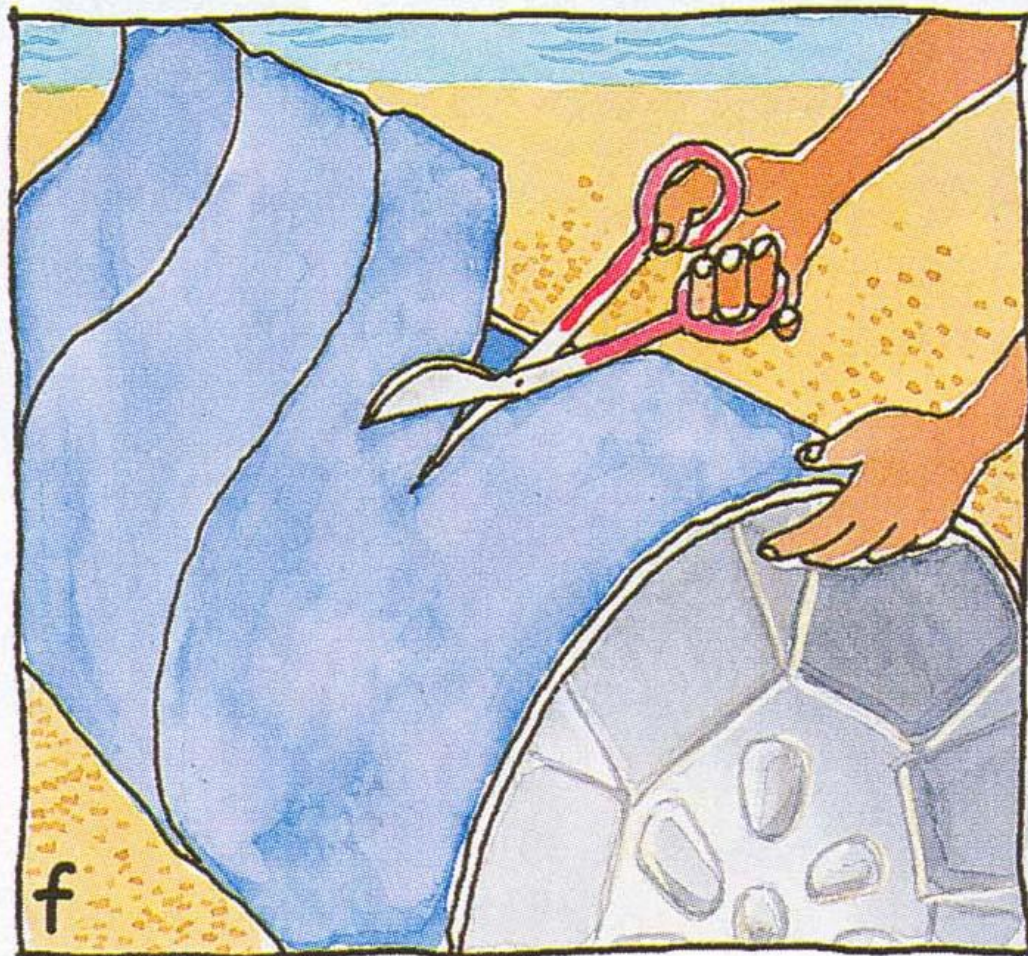


5. Nodeafgrænsning

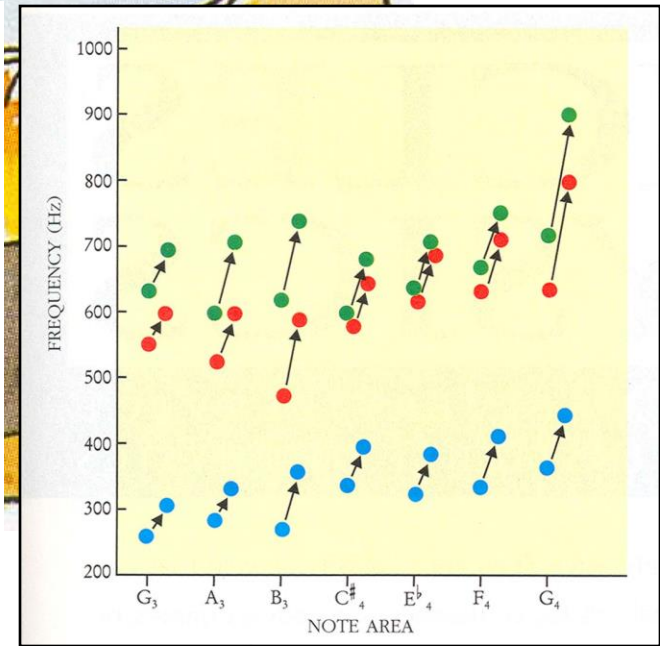
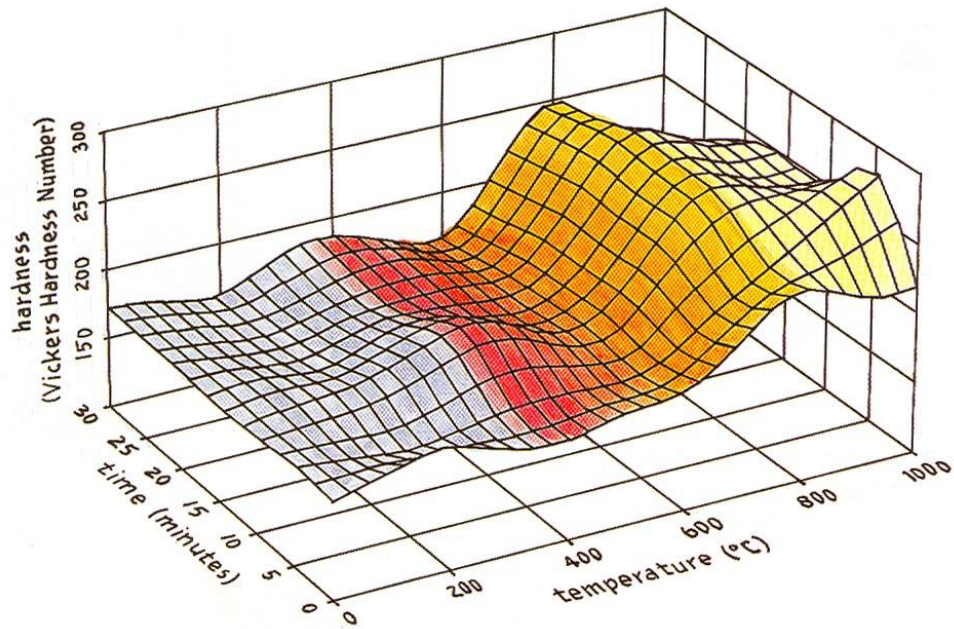
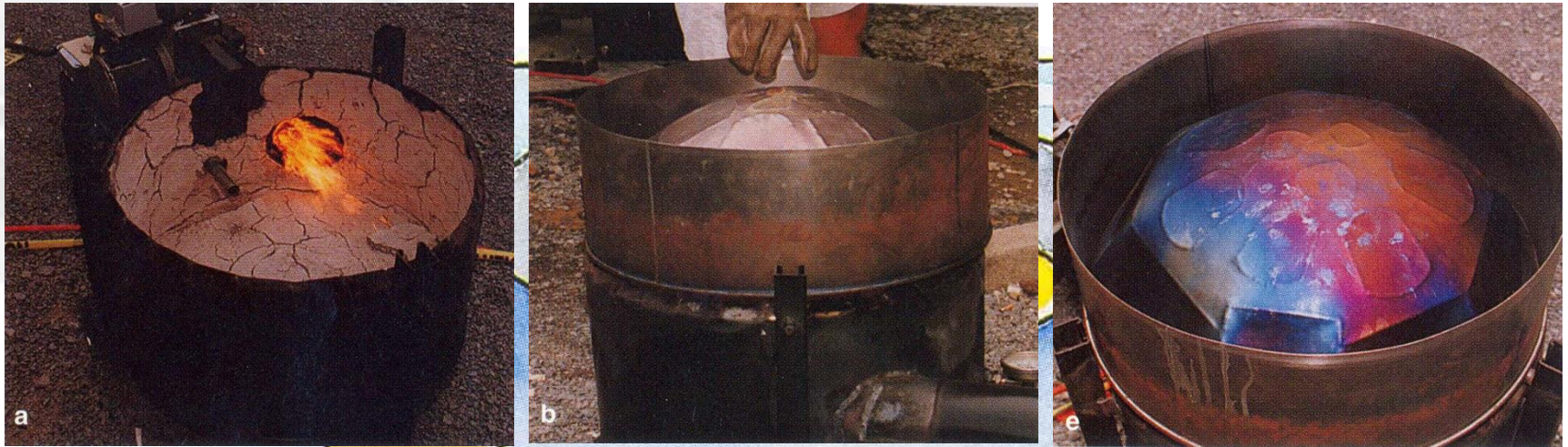


Ref.: [4], [5]

6. Skørtetilpasning

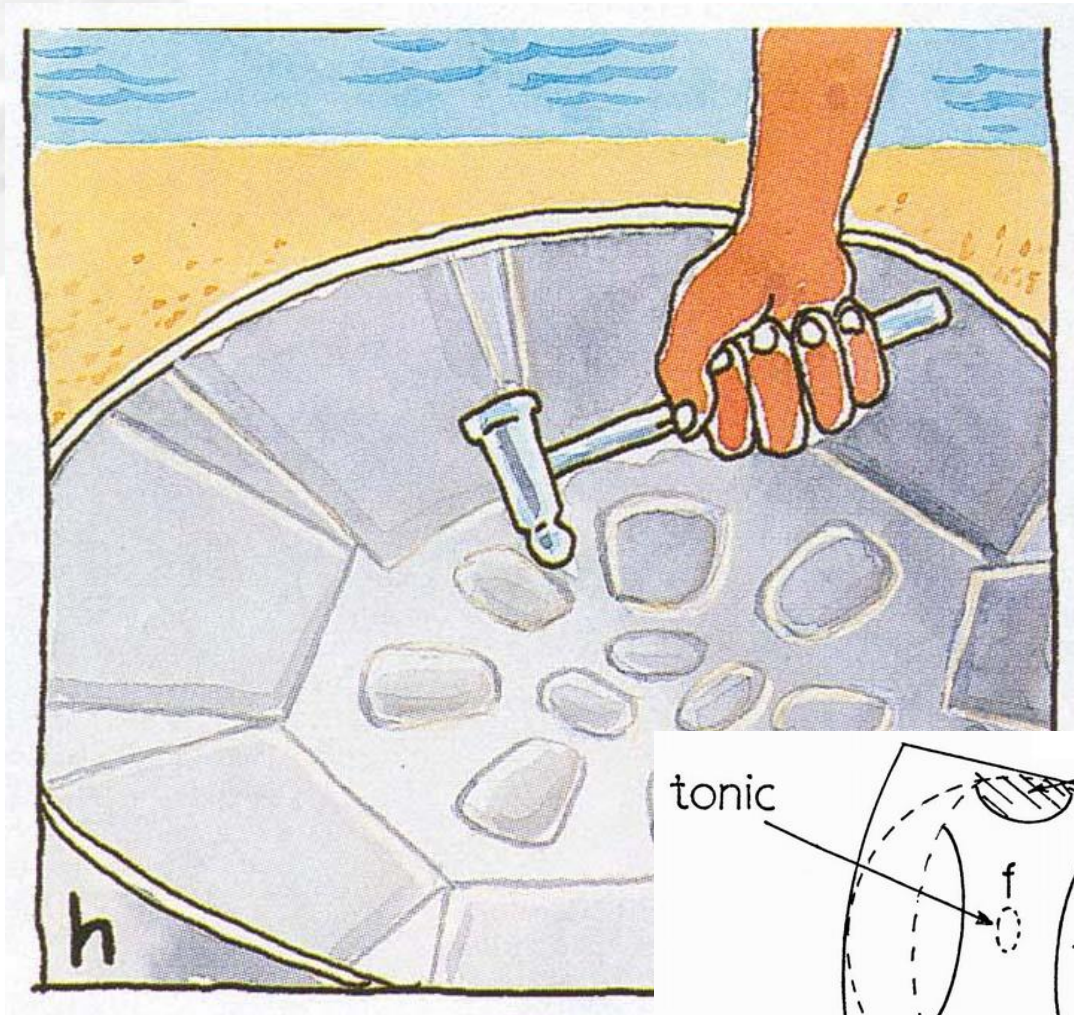


7. Varmebehandling



Ref:
[3],
[4]

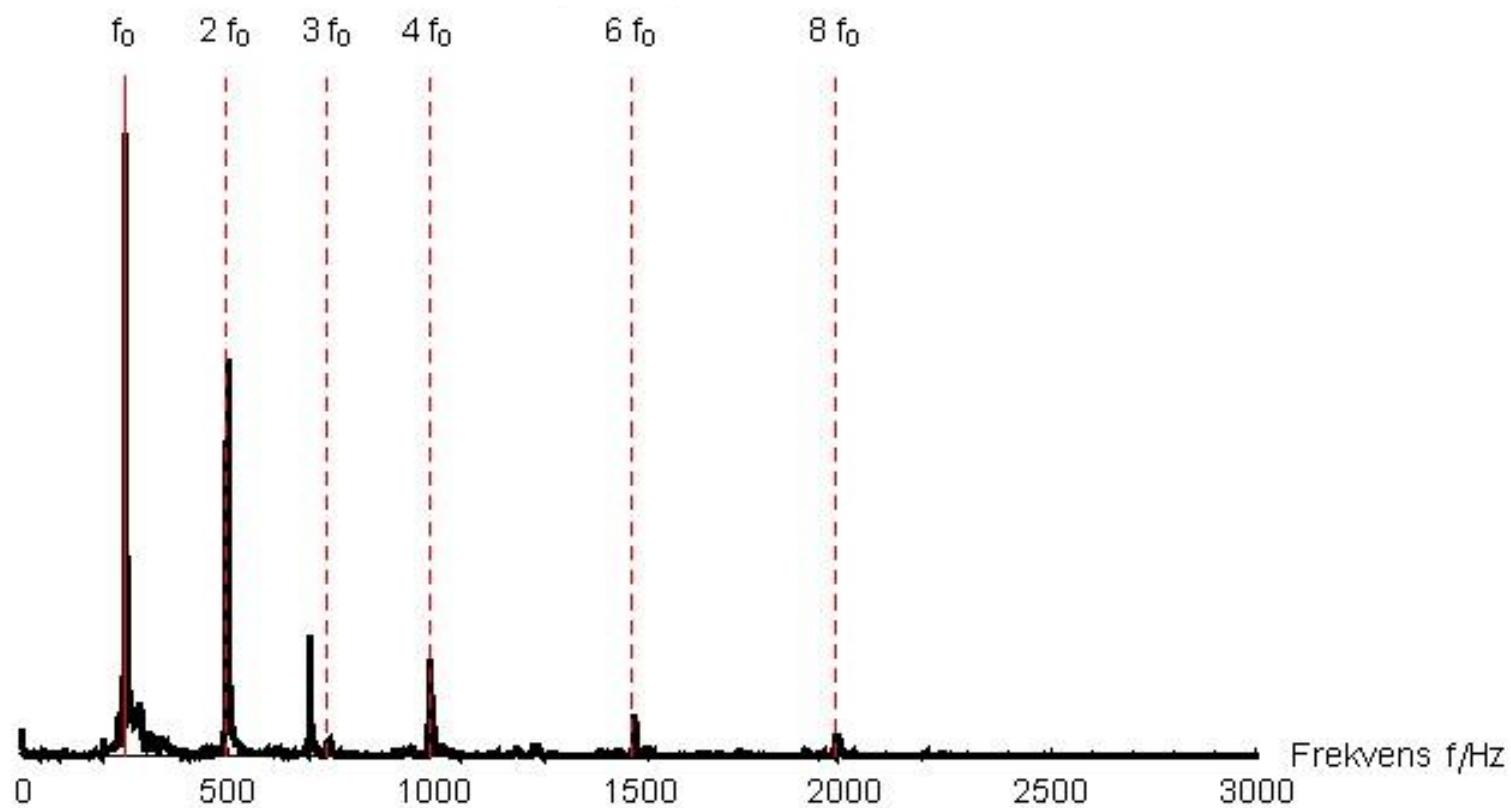
8. Stemming



Ref.: [4], [5]

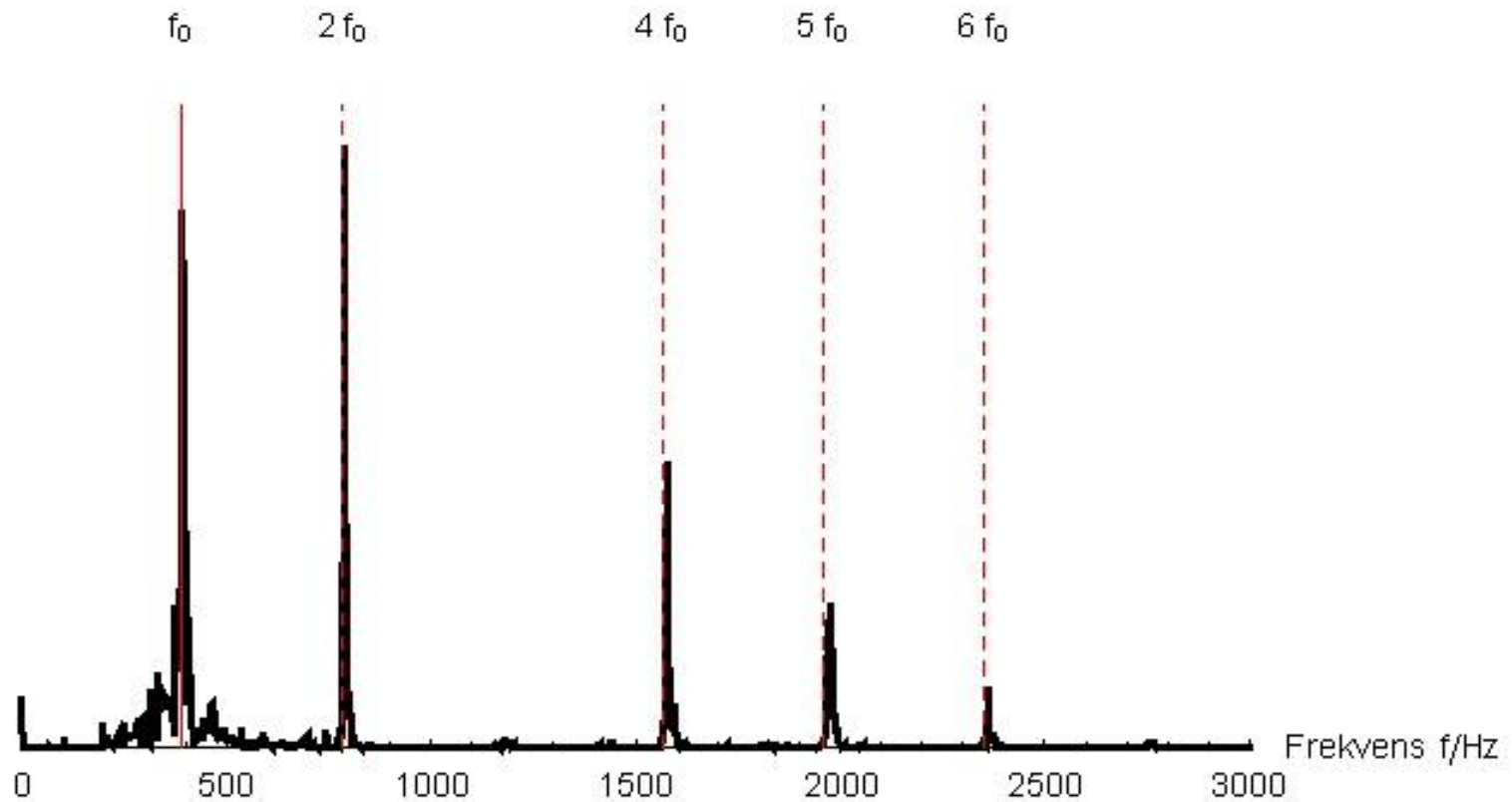
Lydspektre

$$H_3 = 246,9 \text{ Hz}$$



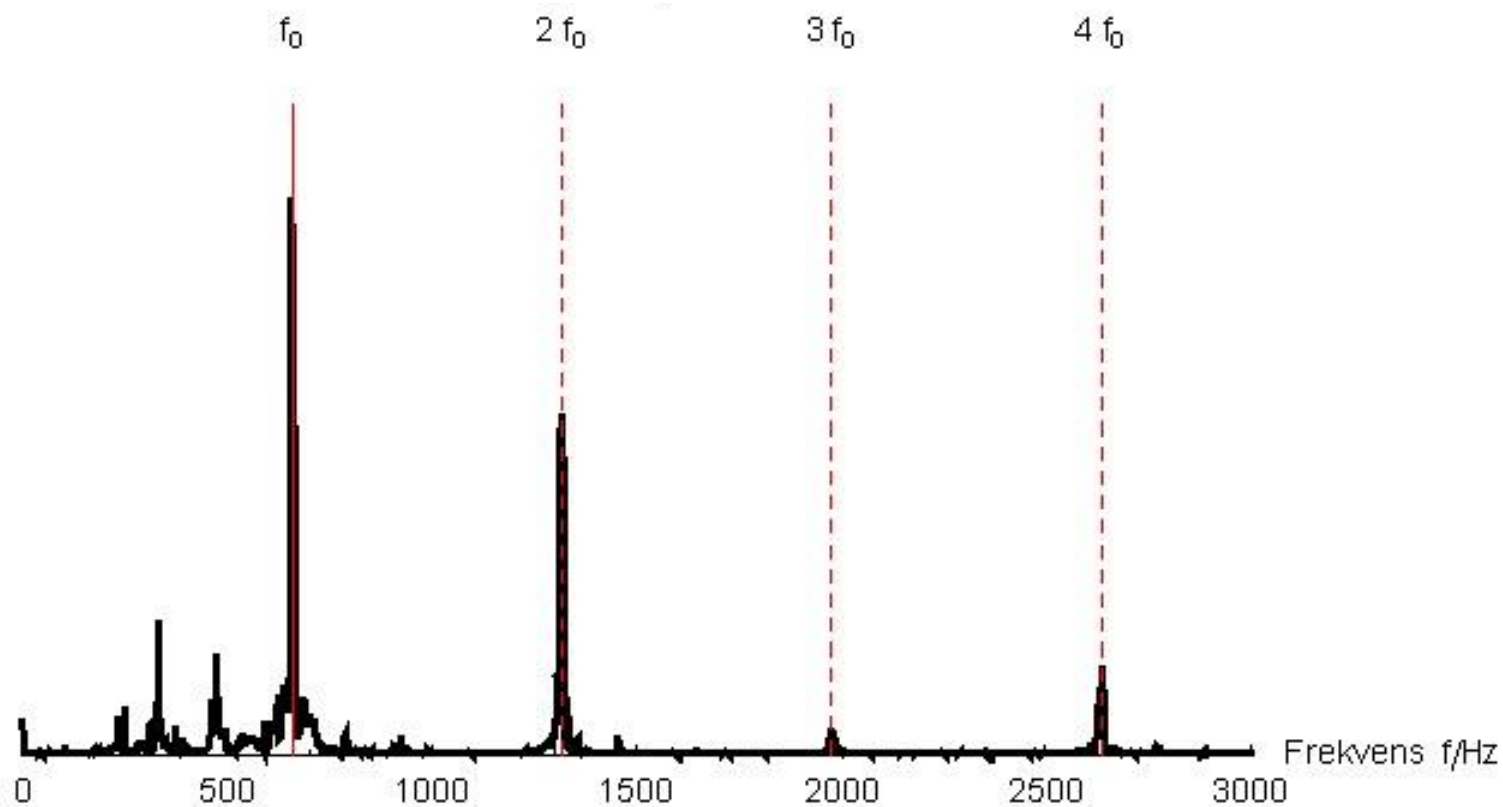
Lydspektre

$$G_4 = 392,0 \text{ Hz}$$



Lydspektre

$$E_5 = 659,3 \text{ Hz}$$





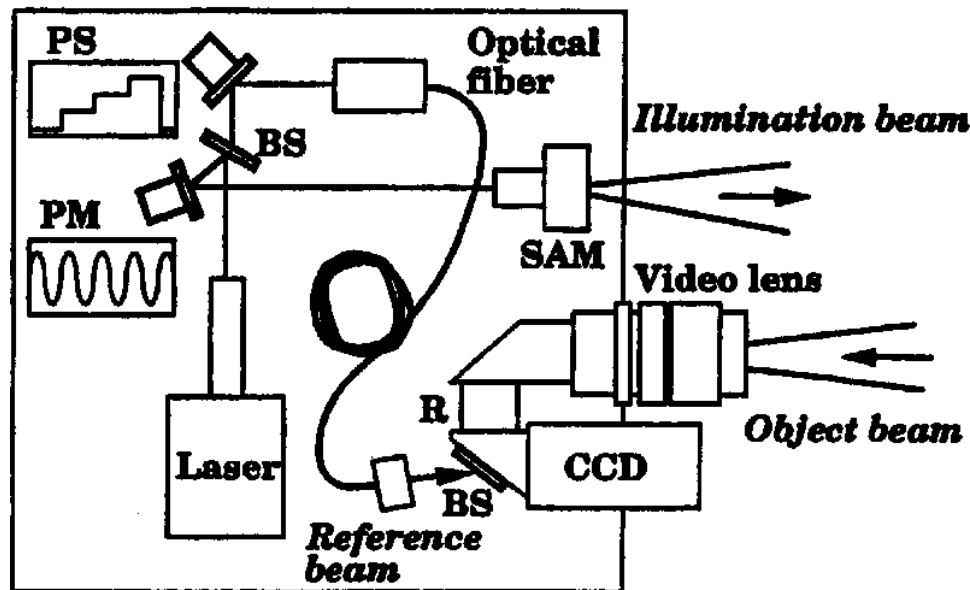
Lydspektre

De mange harmoniske overtoner stammer fra tre fysiske fænomener:

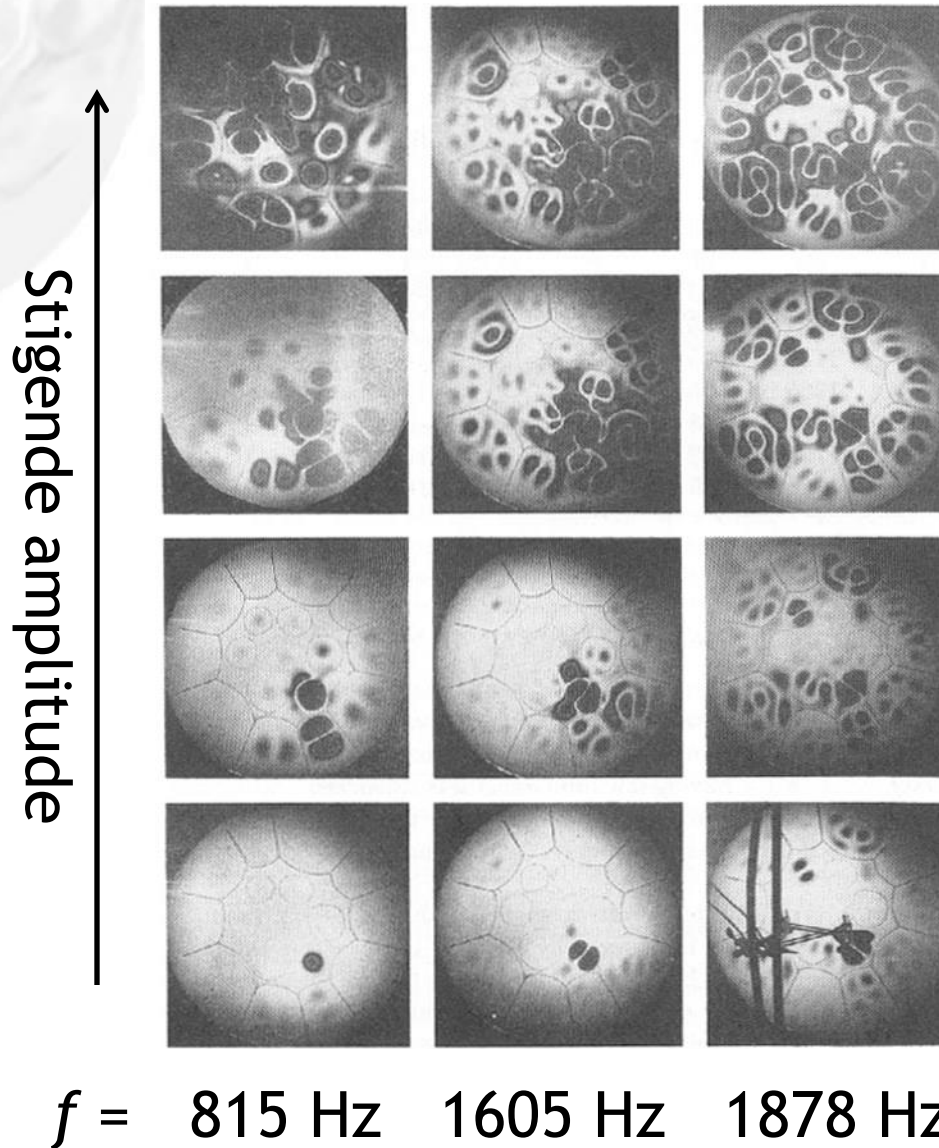
- Lydudstråling fra højere ordens normalsvingninger, som er blevet harmonisk stemt.
- Lydudstråling fra nabonoder, som står i et harmonisk forhold til den anslåede node (sympatisk vibration).
- Ikke-lineær bevægelse af nodeområdet ved grundfrekvensen.

Holografisk afbildning

Den ikke-lineære kobling mellem nodeområder kan studeres vha. holografisk interferometri.

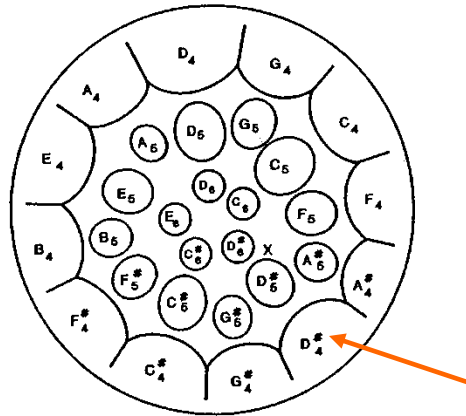


Kobling mellem noder

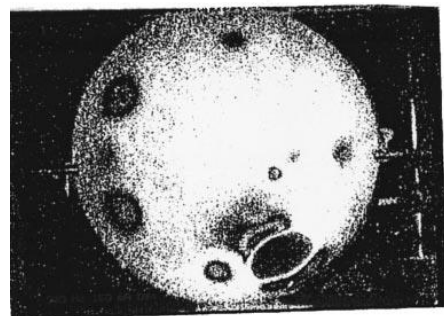


Normalsvingninger

Steelpan design:



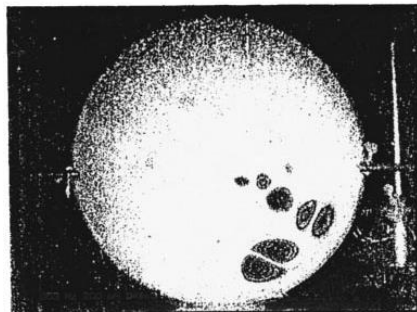
Ref.: [2]



(0,0)



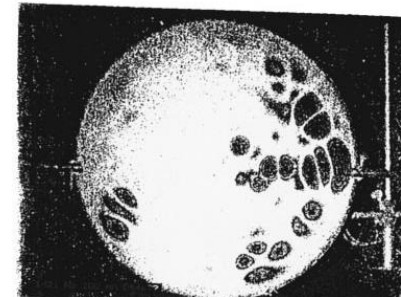
310 Hz
1.00



(1,0)



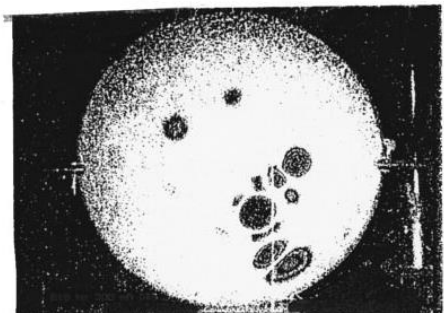
953 Hz
3.06



(0,2 + 2,0)



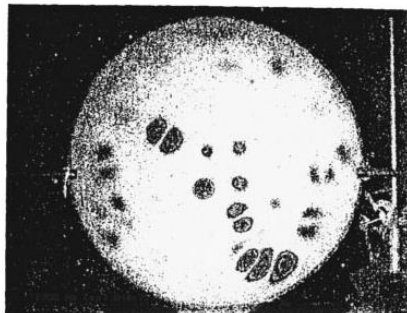
1421 Hz
4.57



(0,1)



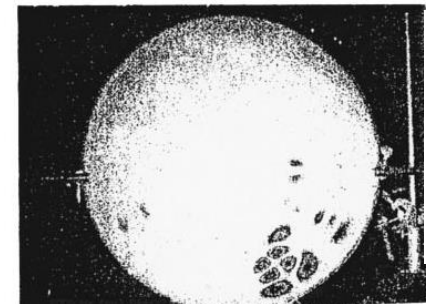
618 Hz
1.99



(0,2)



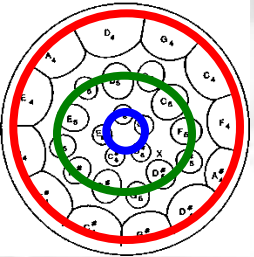
1302 Hz
4.19



(1,2)

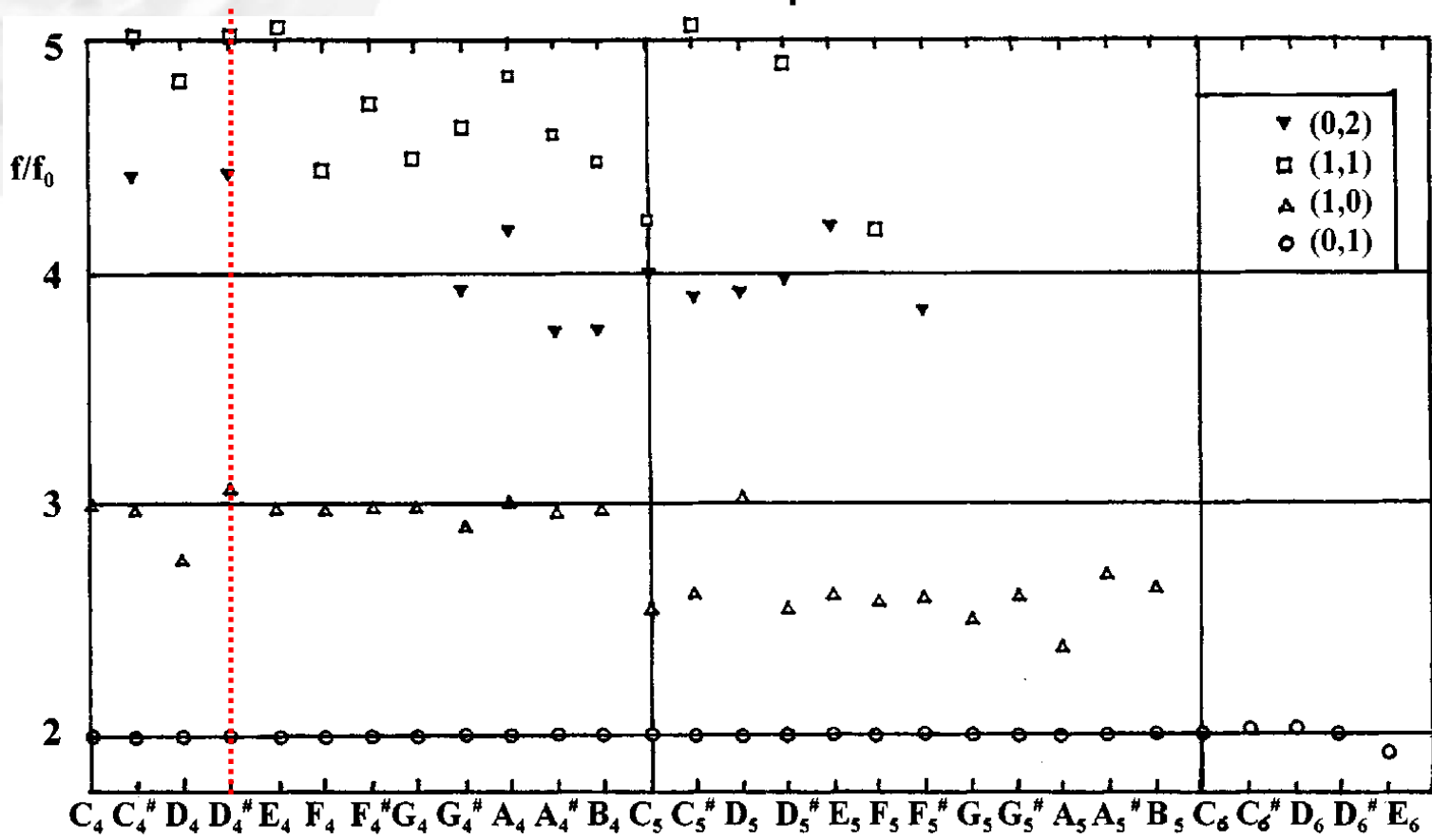


2005 Hz
6.44

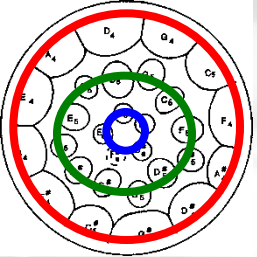


Harmonisk stemning

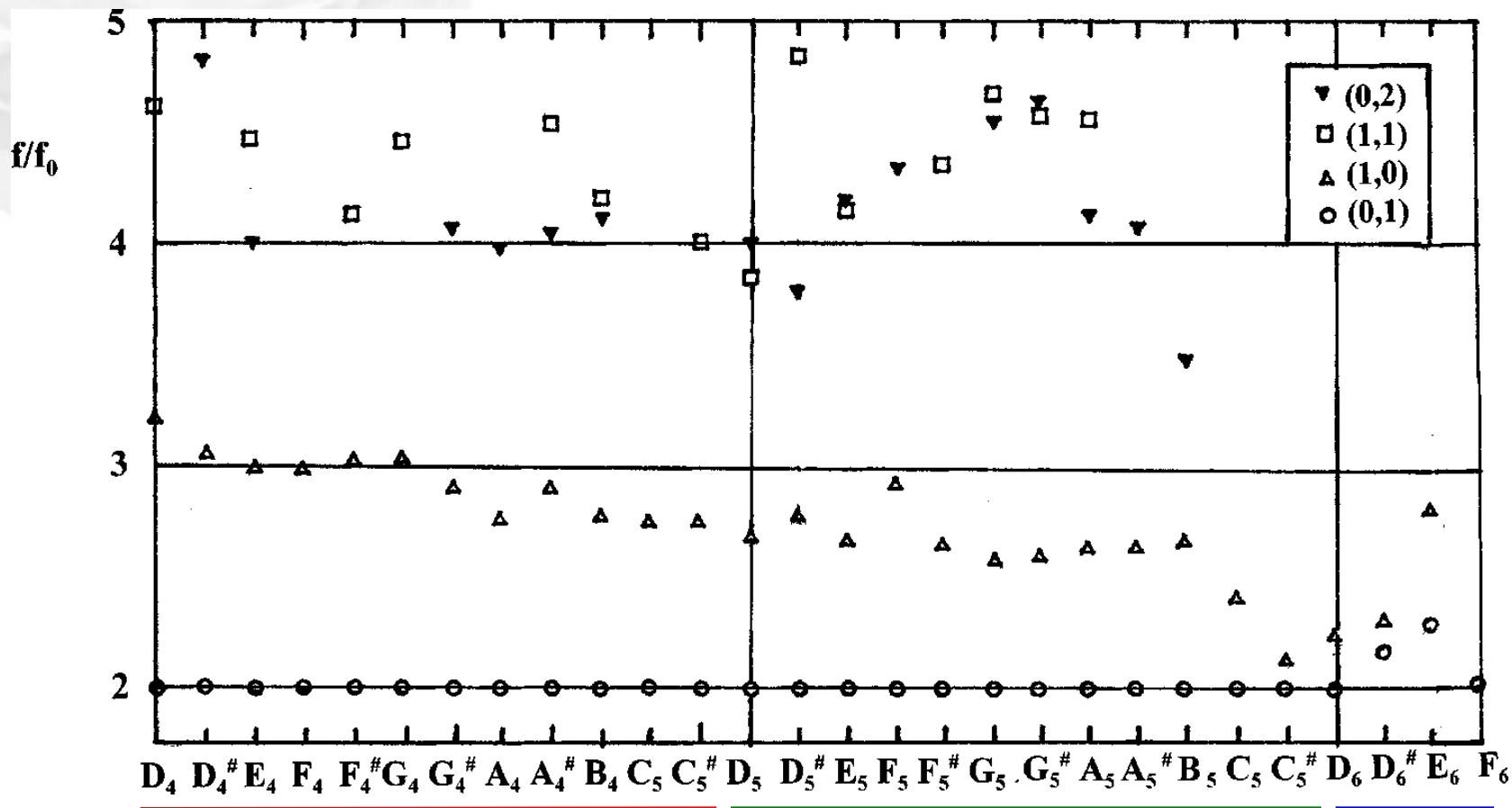
Oversigt over de lavfrekvente normalsvingninger i alle noderne i en tenor steelpan.



Steelpan lavet af Felix Rohner

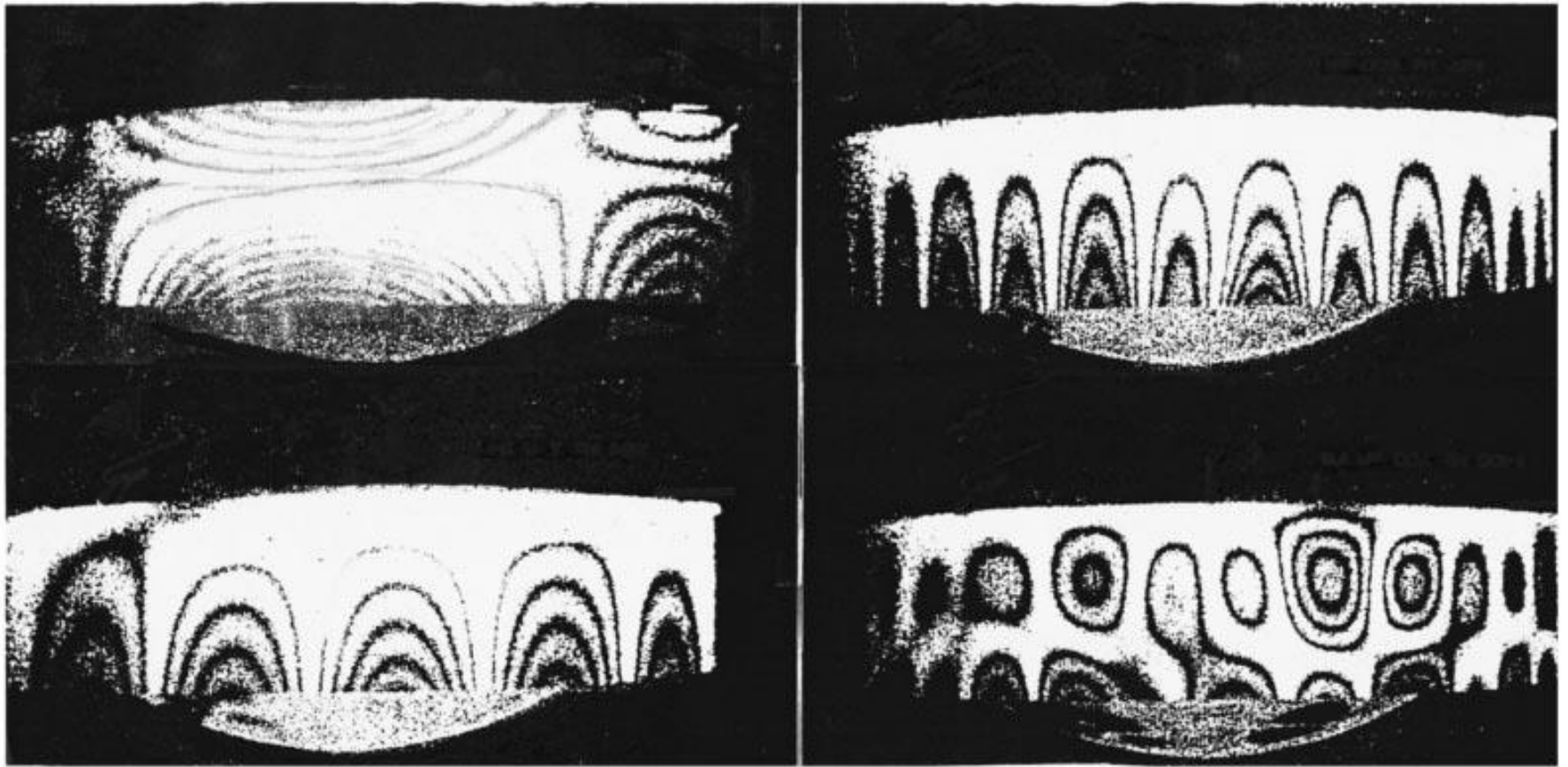


Harmonisk stemning



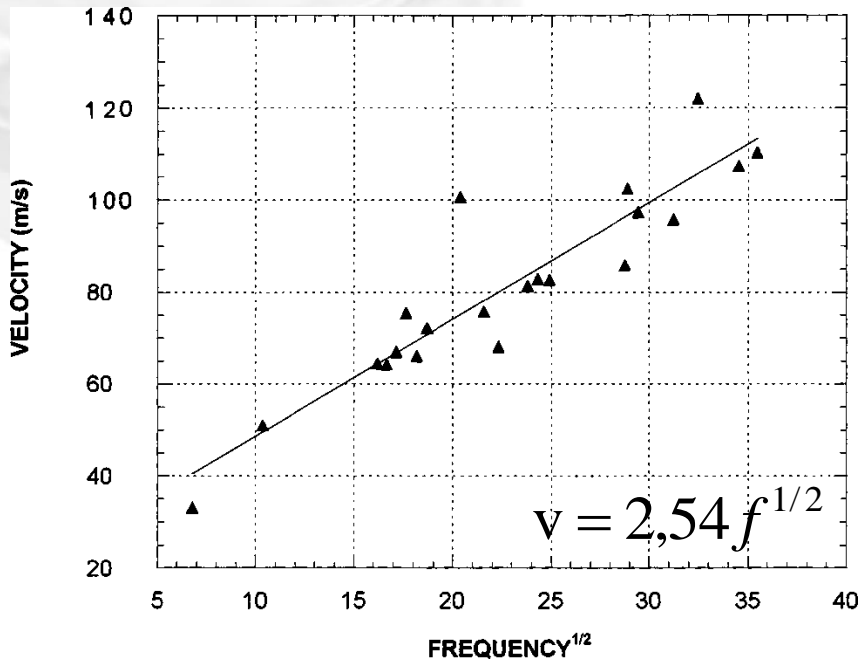
Tenor steelpan lavet af Clifford Alexis

Stående bølger i skørtet

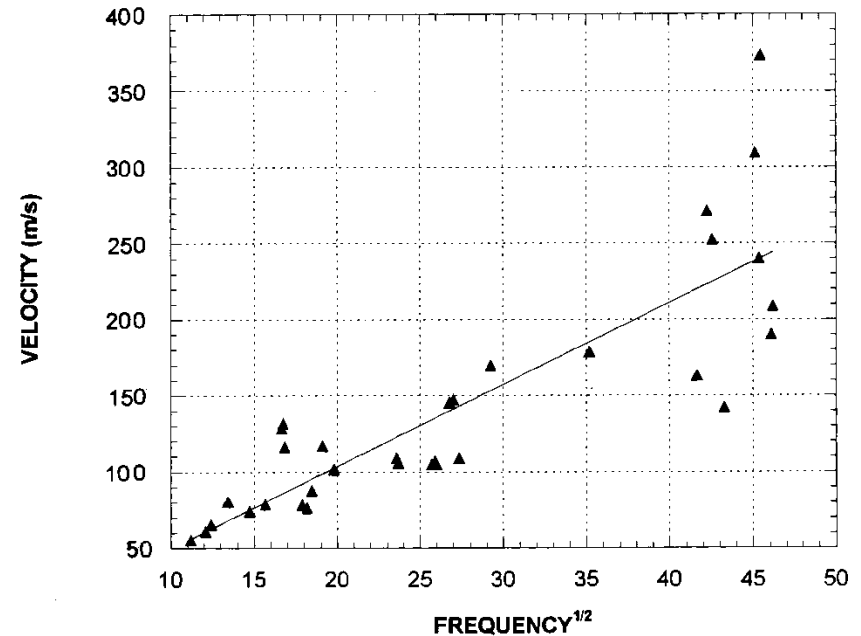


Tenor steelpan lavet af Clifford Alexis

Stående bølger i skørtet



Clifford Alexis' steelpan



Felix Rohners steelpan

For flad, tynd plade:
$$v = \left(\frac{1,8hE}{\rho(1-\nu^2)} f \right)^{1/2} = 2,86 f^{1/2}$$



Demonstration

The Roaring Lion: "Trinidad, the Land of Calypso"
på double tenor steelpans

Referencer

- [1] Fletcher, Rossing: *The Physics of Musical Instruments*, 2. udgave, Springer, New York, 1998.
- [2] Rossing, Hansen, Hampton (2000): Vibrational mode shapes in Caribbean steelpans. I. Tenor and double second, *J. Acoust. Soc. Am.*, **108**(2), 803-811
- [3] Rossing, Hampton, Hansen (1996): Musik from Oil Drums: The Acoustics of the Steel Pan, *Physics Today*, **49**(3), 24-29
- [4] Murr, Ferreyra (2000): Connecting Materials Science and Music in Steel Drums, *American Scientist*, **88**(1), 38-45
- [5] Murr, Ferreyra *et al.* (1999): Materials science and metallurgy of the Caribbean steel drum. Part I. Fabrication, deformation phenomena and acoustic fundamentals. *J. Mater. Sci.*, **34**, 967-979
- [6] Math and Music: A Primer. (Om nodeskalaen)
www.members.cox.net/mathmistakes/music.htm
- [7] Kreyszig: *Advanced Engineering Mathematics*, 8. udgave, John Wiley & Sons, kapitel 11.

... tak for opmærksomheden